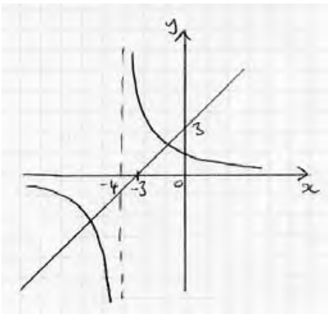
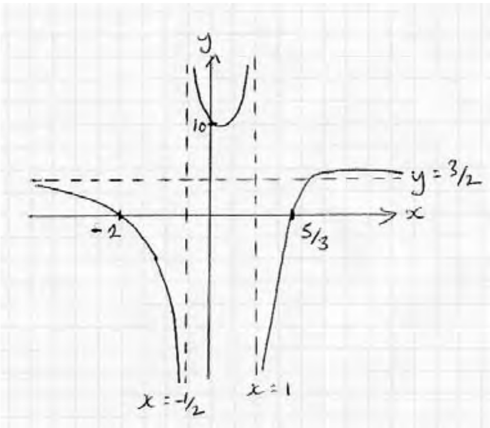
4755 (FP1) Further Concepts for Advanced Mathematics

Section A

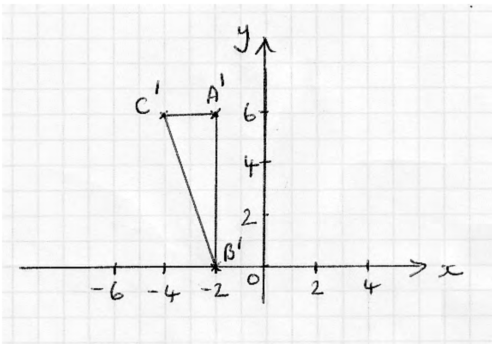
1(i)	$\mathbf{M}^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$	M1 A1 **[2]**	Dividing by determinant
1(ii)	$\frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 49 \\ 100 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 198 \\ 253 \end{pmatrix}$ $\Rightarrow x = 18, y = 23$	M1 A1(ft) A1(ft) **[3]**	Pre-multiplying by their inverse
2	$z^3 + z^2 - 7z - 15 = (z-3)(z^2 + 4z + 5)$ $z^2 + 4z + 5 = 0 \Rightarrow z = \frac{-4 \pm \sqrt{16-20}}{2}$ $\Rightarrow z = -2 + \text{j}$ and $z = -2 - \text{j}$	B1 M1 A1 M1 A1 **[5]**	Show z = 3 is a root; may be implied Attempt to find quadratic factor Correct quadratic factor Use of quadratic formula or other valid method Both solutions
3(i)		B1 B1 **[2]**	Asymptote at x = -4 Both branches correct
3(ii)	$\frac{2}{x+4} = x + 3 \Rightarrow x^2 + 7x + 10 = 0$ $\Rightarrow x = -2$ or $x = -5$ $x \geq -2$ or $-4 > x \geq -5$	M1 A1 A1 A2 **[5]**	Attempt to find where graphs cross or valid attempt at solution using inequalities Correct intersections (both), or -2 and -5 identified as critical values $x \geq -2$ $-4 > x \geq -5$ s.c. A1 for $-4 \geq x \geq -5$ or $-4 > x > -5$

4	$2w-6w+3w=\frac{-1}{2}$ $\Rightarrow w=\frac{1}{2}$ $\Rightarrow$ roots are $1, -3, \frac{3}{2}$ $\frac{-q}{2}=\alpha\beta\gamma=\frac{-9}{2}\Rightarrow q=9$ $\frac{p}{2}=\alpha\beta+\alpha\gamma+\beta\gamma=-6\Rightarrow p=-12$	M1 A1 A1 M1 A2(ft) **[6]**	Use of sum of roots – can be implied Correct roots seen Attempt to use relationships between roots s.c. M1 for other valid method One mark each for *p* = -12 and *q* = 9

5(i)	$\frac{1}{5r-2}-\frac{1}{5r+3}\equiv\frac{5r+3-5r+2}{(5r+3)(5r-2)}$ $\equiv\frac{5}{(5r+3)(5r-2)}$	M1 A1 **[2]**	Attempt to form common denominator Correct cancelling
5(ii)	$\sum_{r=1}^{30}\frac{1}{(5r-2)(5r+3)}=\frac{1}{5}\sum_{r=1}^{30}\left[\frac{1}{(5r-2)}-\frac{1}{(5r+3)}\right]$ $=\frac{1}{5}\left[\left(\frac{1}{3}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{18}\right)+\ldots+\left(\frac{1}{5n-7}-\frac{1}{5n-2}\right)+\left(\frac{1}{5n-2}-\frac{1}{5n+3}\right)\right]$ $=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5n+3}\right]=\frac{n}{3(5n+3)}$	B1 B1 M1 A1 **[4]**	First two terms in full Last term in full Attempt to cancel terms
6	When n = 1, $\frac{1}{2}n(7n-1)=3$, so true for n = 1 Assume true for $n = k$ $3+10+17+.....+(7k-4)=\frac{1}{2}k(7k-1)$ $\Rightarrow 3+10+17+.....+(7(k+1)-4)$ $=\frac{1}{2}k(7k-1)+(7(k+1)-4)$ $=\frac{1}{2}[k(7k-1)+(14(k+1)-8)]$ $=\frac{1}{2}[7k^2+13k+6]$ $=\frac{1}{2}(k+1)(7k+6)$ $=\frac{1}{2}(k+1)(7(k+1)-1)$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for n = 1, it is true for n = 1, 2, 3 and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for $n = k$ Add $(k+1)$th term to both sides Valid attempt to factorise c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
7(i)	$(0, 10), (-2, 0), \left(\frac{5}{3}, 0\right)$	B1 B1 B1 [3]	
7(ii)	$x = \frac{-1}{2}, \ x = 1, \ y = \frac{3}{2}$	B1 B1 B1 [3]	
7(iii)	Large positive *x*, $y \to \frac{3}{2}^{+}$ (e.g. consider $x = 100$) Large negative *x*, $y \to \frac{3}{2}^{-}$ (e.g. consider $x = -100$)	M1 B1 B1 [3]	Clear evidence of method required for full marks
7(iv)	Curve 3 branches of correct shape Asymptotes correct and labelled Intercepts correct and labelled 	B1 B1 B1 [3]	

8 (i)	$\lvert z-(4+2\text{j})\rvert=2$	**B1** **B1** **B1** **[3]**	Radius = 2 $z-(4+2\text{j})$ or $z-4-2\text{j}$ All correct
8(ii)	$\arg\left(z-(4+2\text{j})\right)=0$	**B1** **B1** **B1** **[3]**	Equation involving the argument of a complex variable Argument = 0 All correct
8(iii)	$a=4-2\cos\frac{\pi}{4}=4-\sqrt{2}$ $b=2+2\sin\frac{\pi}{4}=2+\sqrt{2}$ $\text{P}=4-\sqrt{2}+\left(2+\sqrt{2}\right)\text{j}$	**M1** **A2** **[3]**	Valid attempt to use trigonometry involving $\frac{\pi}{4}$, or coordinate geometry 1 mark for each of *a* and *b* s.c. A1 only for *a* = 2.59, *b* = 3.41
8(iv)	$\frac{3}{4}\pi>\arg\left(z-(4+2\text{j})\right)>0$ and $\lvert z-(4+2\text{j})\rvert<2$	**B1** **B1** **B1** **[3]**	$\arg\left(z-(4+2\text{j})\right)>0$ $\arg\left(z-(4+2\text{j})\right)<\frac{3}{4}\pi$ $\lvert z-(4+2\text{j})\rvert<2$ Deduct one mark if only error is use of inclusive inequalities

Section B (continued)			
9(i)	Matrix multiplication is associative	B1 [1]	
	$\mathbf{MN} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	M1	Attempt to find **MN** or **QM**
	$\Rightarrow \mathbf{MN} = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix}$	A1	or $\mathbf{QM} = \begin{pmatrix} 0 & -2 \\ 3 & 0 \end{pmatrix}$
	$\mathbf{QMN} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$	A1(ft) [3]	
9(ii)	M is a stretch, factor 3 in the *x* direction, factor 2 in the *y* direction.	B1 B1	Stretch factor 3 in the *x* direction Stretch factor 2 in the *y* direction
	N is a reflection in the line *y* = *x*.	B1	
	Q is an anticlockwise rotation through 90° about the origin.	B1 [4]	
9(iii)	$\begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -4 \\ 6 & 0 & 6 \end{pmatrix}$	M1 A1(ft)	Applying their **QMN** to points. Minus 1 each error to a minimum of 0.
		B2 [4]	Correct, labelled image points, minus 1 each error to a minimum of 0. Give B4 for correct diagram with no workings.
			Section B Total: 36
			Total: 72